# KINGSWAY CHRISTIAN COLLEGE

## MATHS DEPARTMENT

**Course:** Mathematics Methods Year 12

**Assessment Task:** Test 1 – Exponential Functions & Differentiation

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**Date:** 16th February 2017

**Assessment Score:** _____ / 40

**Year Score:** _____

**Comments:** _____

**Teacher signature:** _____

**Parent/ Guardian signature:** _____

**Comments:** _____

**Question 1:** **[3 Marks]**

The population of a certain fish in the Ningaloo Reef grows continuously at a rate of 5% per year. The number of fish on 1st January, 2016 was estimated at 2500.

a) Find an expression to model P, the number of fish, t years into the study.

$P = P_0 e^{kt}$

$\therefore P = 2500 e^{0.05t}$ ✓

b) Find the population at 1st January, 2020. Give your answer in terms of *e*.

$P = P_0 e^{kt}$ $\quad \therefore t = 4 yrs$

$= 2500 e^{0.05(4)}$

$\therefore P = 2500 e^{0.2}$ ✓

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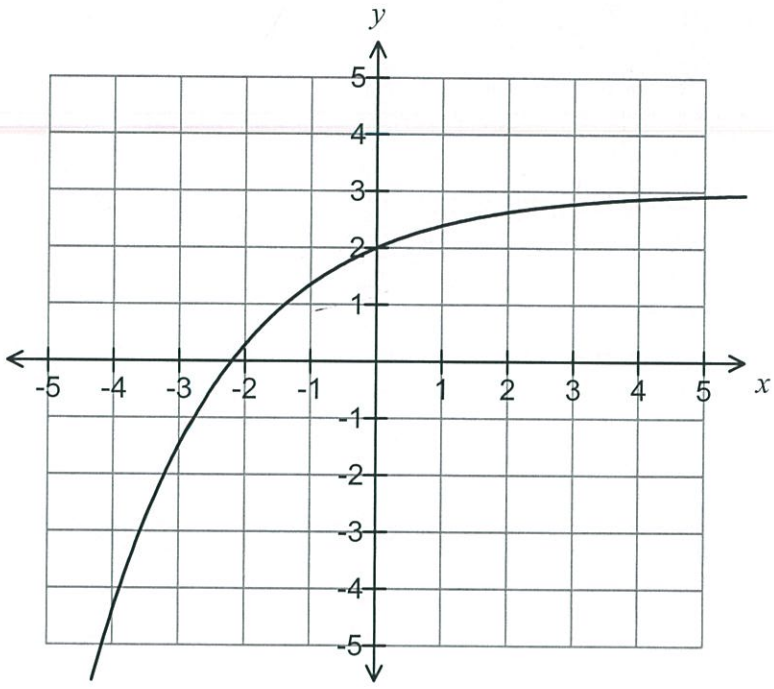
c) Give the calculator algorithm you would use to calculate the time, t, when the population will quadruple in size.

Solve $(10\,000 = 2500 e^{0.05t}, t)$ ✓

**Question 2:** **[2,2 = 4 Marks]**

The graph of $y = ae^{bx} + c$ is shown below. The graph passes through the point $(0, 2)$, and $y \to 3$ as $x \to \infty$.




a) Is $b$ positive or negative? Justify your answer.

Negative ✓ The graph is reflected ✓ in the y-axes (and also in the x-axes). (2)

b) Evaluate $a$ and $c$.

$C = 3$ ✓ $\longrightarrow$

$y = ae^{bx} + 3$

through $(0, 2)$

$2 = a \cdot e^{b \cdot 0} + 3$ (2)

$2 = a \cdot 1 + 3$

$-1 = a$ $\longleftarrow$

$\therefore a = -1$ ✓ $\longrightarrow$

**Question 3:** **[3, 2 = 5 Marks]**

Find $\frac{dy}{dx}$ if:

a) $y = \frac{\sqrt[3]{x^2} - 6x^2}{2x}$

$\therefore y = \frac{x^{\frac{2}{3}}}{2x} - \frac{6x^2}{2x}$ ✓

$\therefore y = \frac{x^{-\frac{1}{3}}}{2} - 3x$

$\therefore \frac{dy}{dx} = -\frac{1}{3}\left(\frac{1}{2}x^{-4/3}\right) - 3$ ✓

$\therefore \frac{dy}{dx} = -\frac{1}{6\sqrt[3]{x^4}} - 3$ ✓

(3)

b) $y = 2ax^a - 4a^2$, where $a$ is constant and $a > 0$

$\therefore \frac{dy}{dx} = 2a^2 x^{a-1}$ ✓ ✓

(2) *(-1 if $-8a$ as well.)

**Question 4:** **[2, 2, 3, 3, 3 = 13 Marks]**

Find the derivative of each of the following. Simplify all answers.

a) $y = (2x - 5)(x^2 - 3x + 4)$

$\frac{dy}{dx} = 2(x^2 - 3x + 4) + (2x - 5)(2x - 3)$

$= 2x^2 - 6x + 8 + 4x^2 - 16x + 15$

$\therefore \frac{dy}{dx} = 6x^2 - 22x + 23$ ✓✓

(2)

b) $y = \frac{3x-2}{3x^2+1}$

$\therefore \frac{dy}{dx} = \frac{3(3x^2+1) - (3x-2)(6x)}{(3x^2+1)^2}$

$= \frac{9x^2 + 3 - 18x^2 + 12x}{(3x^2+1)^2} = \frac{-9x^2 + 12x + 3}{(3x^2+1)^2}$ ✓

(2) ✓

c) $(\sqrt[4]{x^2+4})^3$

$$\therefore y = (x^2+4)^{\frac{3}{4}}$$

$$\therefore \frac{dy}{dx} = \frac{3}{4}(x^2+4)^{-\frac{1}{4}} \cdot 2x \quad \checkmark$$

$$= \frac{6x}{4 \cdot (x^2+4)^{\frac{1}{4}}}$$

$$= \frac{3x}{2\sqrt[4]{(x^2+4)}} \quad \checkmark \quad \checkmark$$

(3)

d) $y = \frac{3x^5}{e^{2x}}$

* correct use of quotient rule.

$$\therefore \frac{dy}{dx} = \frac{15x^4 \cdot e^{2x} - 3x^5 \cdot e^{2x} \cdot 2}{(e^{2x})^2} \quad \checkmark$$

$$= \frac{e^{2x}(15x^4 - 6x^5)}{(e^{2x})^2}$$

$$\frac{dy}{dx} = \frac{15x^4 - 6x^5}{e^{2x}} \quad \checkmark \quad \checkmark$$

(3)

e) $y = \frac{3}{\sqrt{1+e^{5x}}}$

$$y = 3 \cdot (1+e^{5x})^{-\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = -\frac{3}{2}(1+e^{5x})^{-\frac{3}{2}} \cdot e^{5x} \cdot 5 \quad \checkmark$$

(3)

$$= \frac{-15e^{5x}}{2\sqrt{(1+e^{5x})^3}} \quad \checkmark$$

**Question 5:** **[3,2 = 5 Marks]**

Differentiate the following, without simplifying:

a) $y = \frac{x-1}{x^2+4}$

$$\therefore \frac{dy}{dx} = \frac{1(x^2+4) - (x-1)(2x)}{(x^2+4)^2} \quad (3)$$

b) $y = e^{2x-x^2}$

$$\therefore \frac{dy}{dx} = e^{2x-x^2} \cdot (2-2x) \quad (2)$$

(or)

$$2 \cdot e^{2x-x^2} \cdot (1-x)$$

$$= 2(1-x) \cdot e^{2x-x^2}$$

**Question 6:** **[4 Marks]**

Show that $y = \frac{1+e^{3x-1}}{2e^{-x^2}}$ can be differentiated **without** using the product **or** quotient rule.

$$\therefore y = \frac{1}{2e^{-x^2}} + \frac{e^{3x-1}}{2e^{-x^2}}$$

$$\therefore y = \frac{1e^{x^2}}{2} + \frac{1e^{x^2+3x-1}}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot e^{x^2} \cdot 2x + \frac{1}{2} e^{x^2+3x-1} \cdot (2x+3)$$

$$= xe^{x^2} + xe^{x^2+3x-1} + \frac{3e^{x^2+3x-1}}{2}$$

$$= xe^{x^2} + \left(x+\frac{3}{2}\right)e^{x^2+3x-1}$$

**Question 7:** **[2, 4 = 6 Marks]**

a) Simplify $y = \frac{4x+12}{x^2-9}$, stating any exclusions from the domain.

$y = \frac{4(x+3)}{(x+3)(x-3)} = \frac{4(x-3)^{-1}}{}$ ; $x \neq \pm 3$.

(OR)

$= \frac{4}{x-3}$

Hence, make use of the chain rule with Leibnitz notation, to determine:

b) $\frac{dz}{dy}$, if $z = \frac{1}{3x}$ and $y = \frac{4x+12}{x^2-9}$

$\therefore z = \frac{1}{3}x^{-1}$

$\therefore \frac{dz}{dx} = -\frac{1}{3x^2}$

From (a)

$\therefore y = \frac{4}{x-3}$

$= 4(x-3)^{-1}$

$\therefore \frac{dy}{dx} = -4(x-3)^{-2} \cdot 1.$

$\therefore \frac{dy}{dx} = -\frac{4}{(x-3)^2}$

then $\frac{dx}{dy} = \frac{(x-3)^2}{-4}$

$\therefore \frac{dz}{dy} = \frac{dz}{dx} \times \frac{dx}{dy}.$

$= -\frac{1}{3x^2} \times \frac{(x-3)^2}{-4}$

$\therefore \frac{dz}{dy} = \frac{(x-3)^2}{12x^2}$

EXTRA WORK SPACE